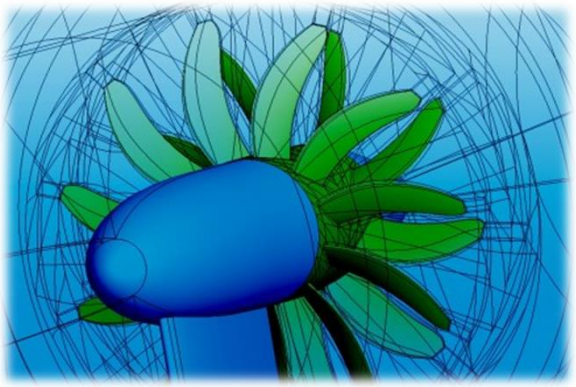
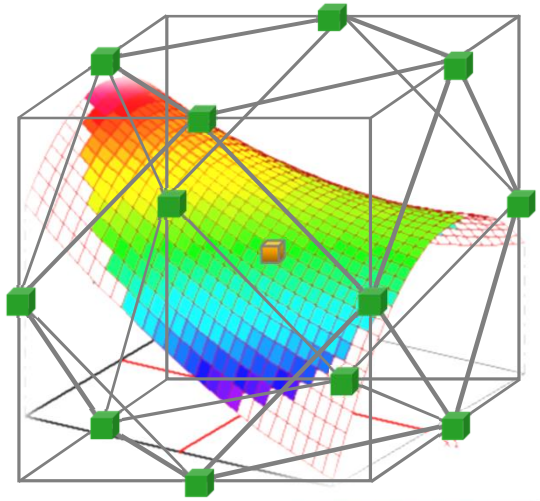


Air TN-NextGEn Workshop, Amsterdam, May 25th 2016

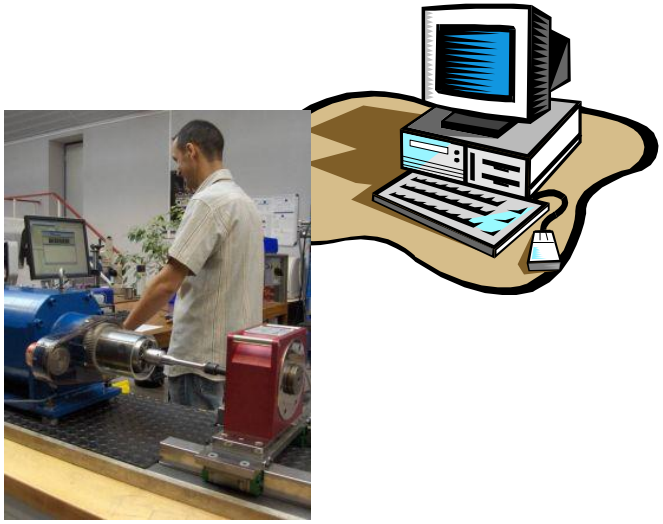
Designs of experiments



Michelle Sergent

Laboratoire d'Instrumentation et Sciences Analytiques. EA 4672

Aix Marseille Université (France)



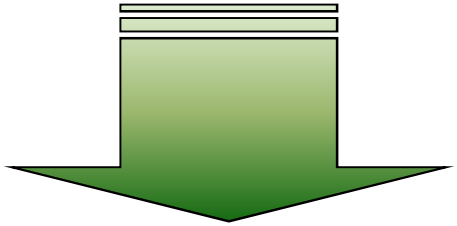
Experiments

Information

Decision

Risk

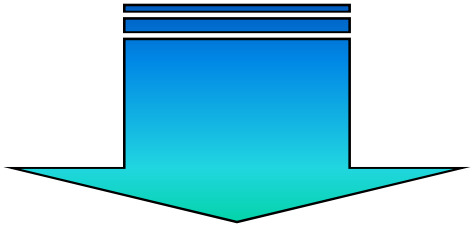
The risk depends on the quality of the information



The information must be of the best quality or, of acceptable quality !!!

The quality of the information does not depend on the number of experiments !

The quality of the information depends on the position of the experimental points



Experimental Planning

Experimental Design can be defined as the strategy for setting up experiments in such a manner that the required information is obtained as efficiently and precisely as possible

Description of the problem

The targets

The list of the **input variables**

The list of the **output variables**

Domain of variation

Elaboration of a strategy

Design of experiments

To elaborate the experimental strategy

to choose an appropriate design of experiments in accordance with

THE OBJECTIVES

Exploratory research
Exploration of the experimental space

Screening of factors
Identification of the influential factors

Quantitative study of factors
Possible interactions between the factors

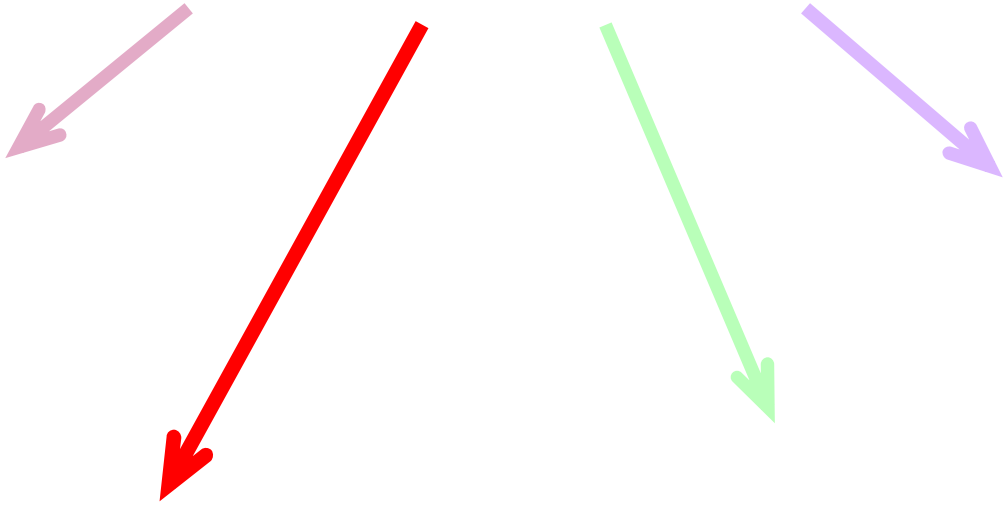
Optimization
Definition of the optimal conditions

Design of experiments

To elaborate the experimental strategy

to choose an appropriate design of experiments in accordance with

THE OBJECTIVES



Screening of factors

Identification of the influential factors

Screening of factors

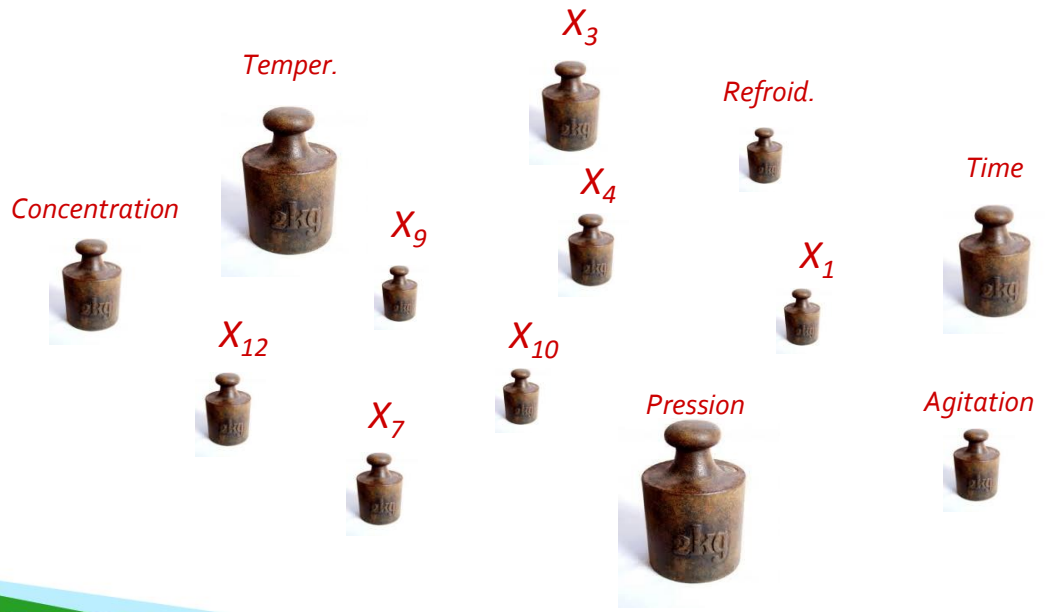
"Scientific" screening

Screening of factors

➡ **Strategy allowing to quickly identify the few really important factors (h) among a lot of potentially influential factors (k)**

h << k

Estimation of the "weight" of the factors



Screening of factors

Example:

Study of a tightness "test bed"

Objective :

The "test bed" are carrefully considered. Every "bed" is tested and calibrated to evaluate their measure uncertainty. Before calculating this uncertainty, we must know the variation sources that are influential on the measure of the friction coefficient.

Identification of the influential factors on the uncertainty of measure

To know the "weight" of each possible source of variation

Example:

FACTORS :

- Threading
- Nut quality
- Degreasing
- Boring
- Length
- Control
- Nut preparation
- Tightening speed
- Amplitude setting
- Programming
- Operator

RESPONSES :

- Friction coefficient

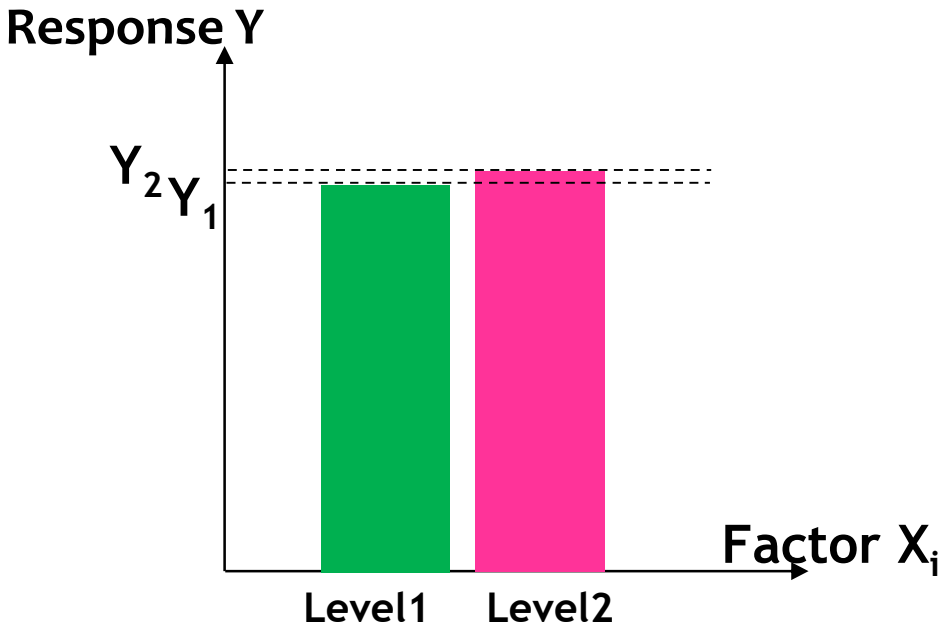
To know the "weight" of each factor

Experimental domain :

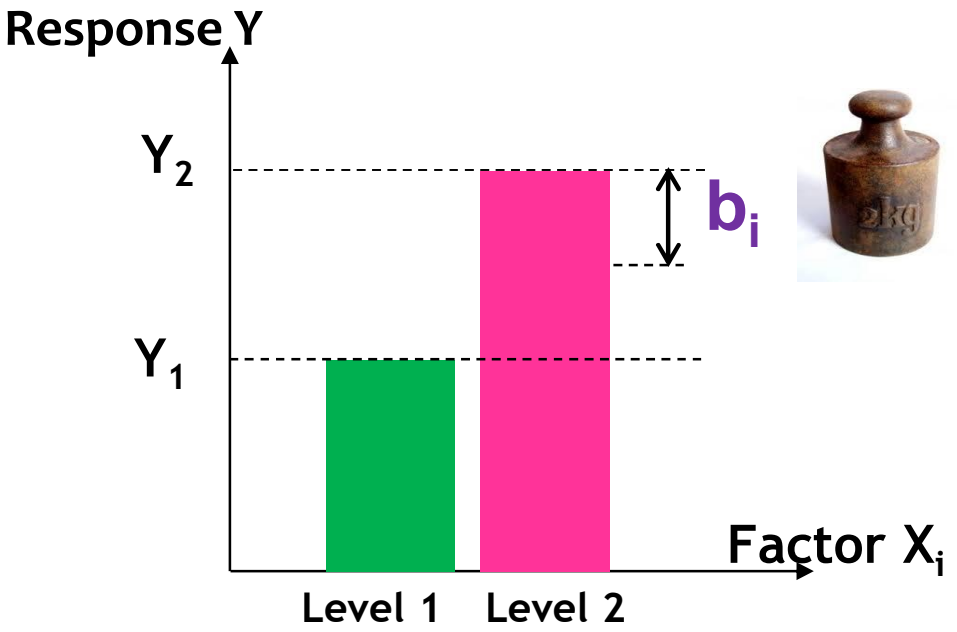
	FACTORS	Level 1 (-)	Level 2 (+)
U_1	Threading	before	after
U_2	Nut quality	10	8
U_3	Degreasing	Ac.Et	Et.pet
U_4	Boring	No	Yes
U_5	Length	30	50
U_6	Control	Yes	No
U_7	Nut preparation	untreated	scrubbed
U_8	Tightening speed	5	12
U_9	Amplitude setting	Regular	Shift
U_{10}	Programming	1	2
U_{11}	Operator	A	B

To know the "weight" of each factor, for these levels

The experimental strategy aims **to link** the variation of the **Responses** (Y_i) to the variation of the **Factors** (X_i)



$b_i \approx 0$

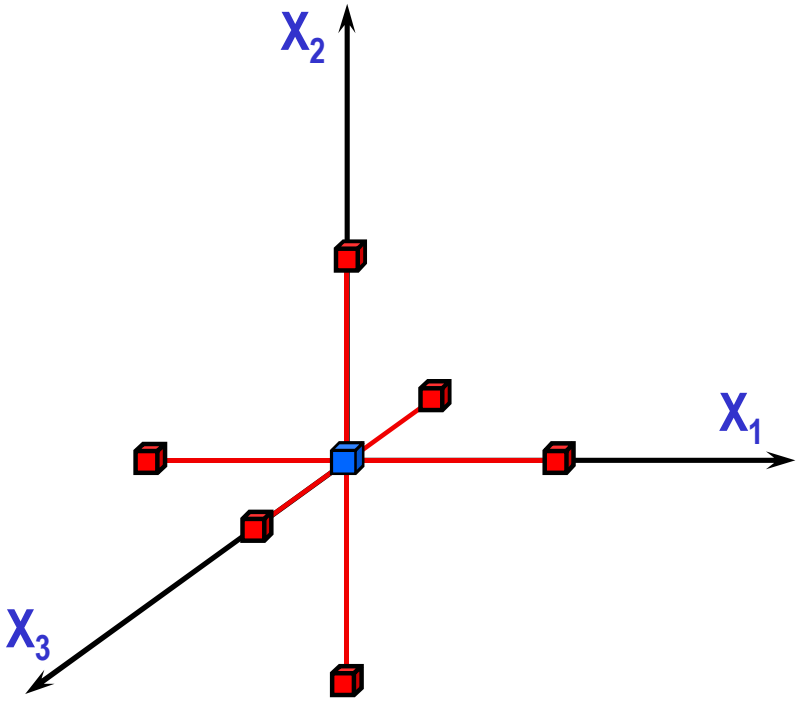


$b_i \neq 0$

b_i : "weight" of the factor X_i

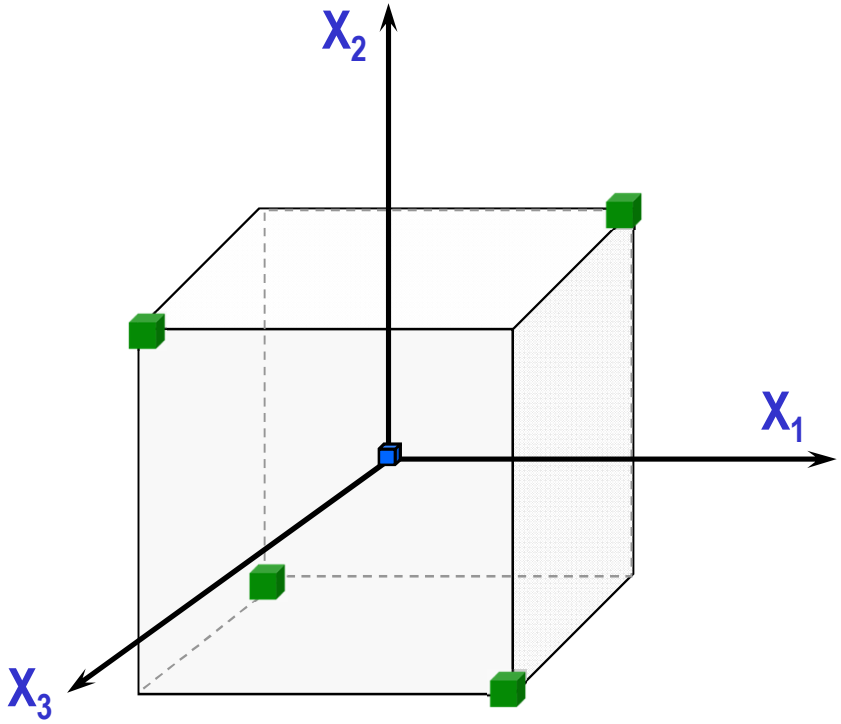
Example *: 3 factors (X_1, X_2, X_3)*

"1 Factor At a Time"



6 experiments

"Experimental design"



4 experiments

b_i : "Effect" of X_1, X_2 and X_3

Optimal experimental strategy

Experimental design to "weigh"
11 factors with 2 levels,
with an optimal quality and a minimum number of experiments

Optimal design of experiments

➡ 12 experiments

b_i : "weight" of the factors

(b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8 , b_9, b_{10} and b_{11})

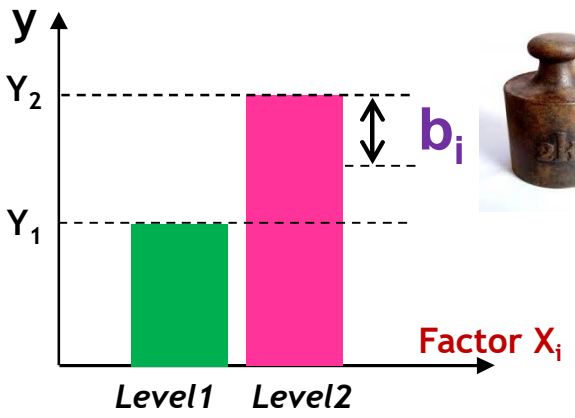
Screening design $2^{11}//12$

N°	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	X_9	X_{10}	X_{11}
1	+	+	-	+	+	+	-	-	-	+	-
2	-	+	+	-	+	+	+	-	-	-	+
3	+	-	+	+	-	+	+	+	-	-	-
4	-	+	-	+	+	-	+	+	+	-	-
5	-	-	+	-	+	+	-	+	+	+	-
6	-	-	-	+	-	+	+	-	+	+	+
7	+	-	-	-	+	-	+	+	-	+	+
8	+	+	-	-	-	+	-	+	+	-	+
9	+	+	+	-	-	-	+	-	+	+	-
10	-	+	+	+	-	-	-	+	-	+	+
11	+	-	+	+	+	-	-	-	+	-	+
12	-	-	-	-	-	-	-	-	-	-	-

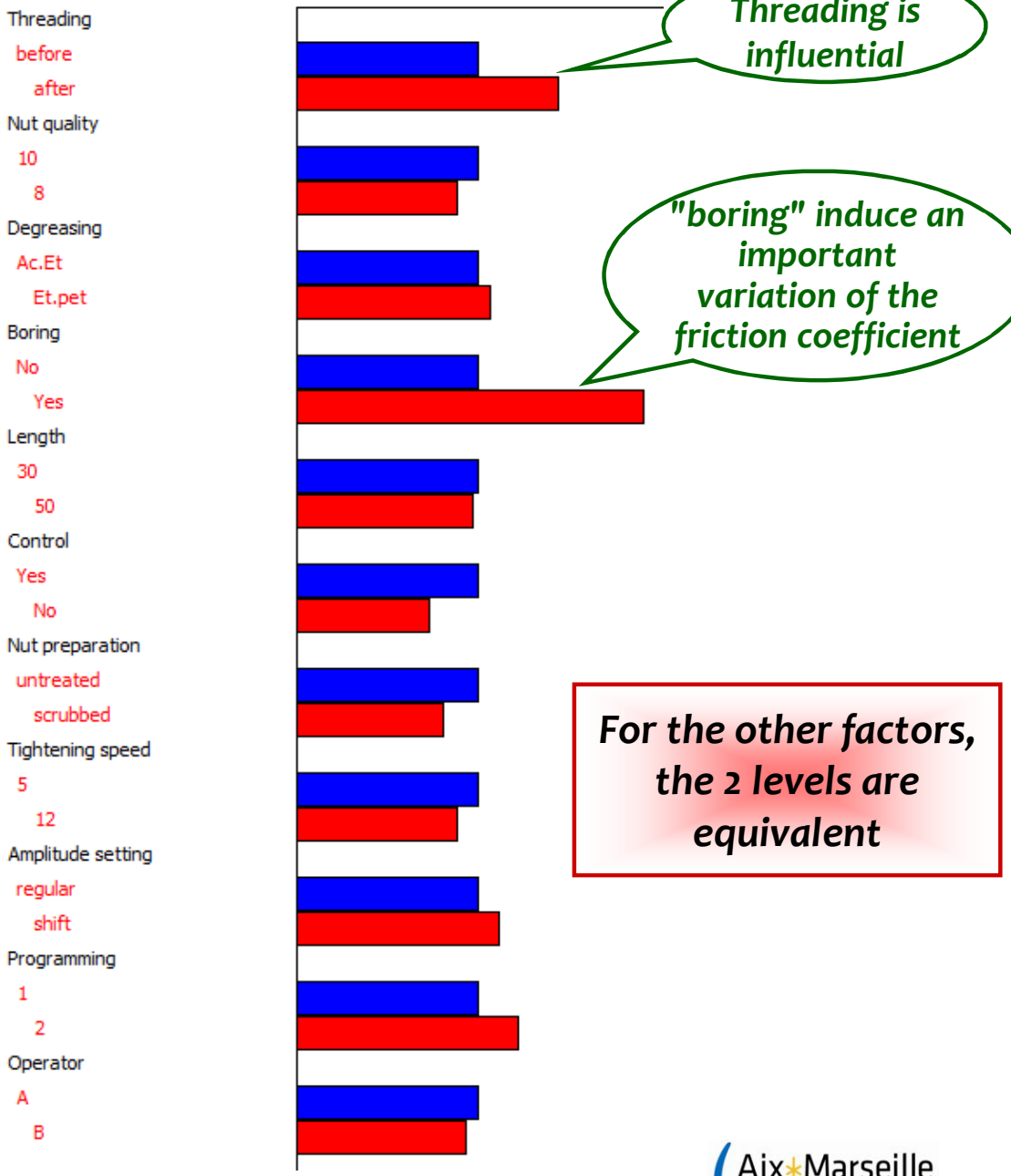
Experimentation

N	U_1	U_2	U_3	U_4	U_5	U_6	U_7	U_8	U_9	U_{10}	U_{11}	Coeff.
1	after	10	Ac.Et	yes	50	no	untreated	5	regular	2	A	1.178
2	before	10	Et.pet	no	50	no	scrubbed	5	regular	1	B	1.139
3	after	8	Et.pe	yes	30	no	scrubbed	12	regular	1	A	1.171
4	before	10	Ac.Et	yes	50	yes	scrubbed	12	shift	1	A	1.165
5	before	8	Et.pet	no	50	no	untreated	5	shift	2	A	1.152
6	before	8	Ac.Et	yes	30	no	scrubbed	12	shift	2	B	1.168
7	after	8	Ac.Et	no	50	yes	scrubbed	12	regular	2	B	1.158
8	after	10	Ac.Et	no	30	no	untreated	12	shift	1	B	1.152
9	after	10	Et.pet	no	30	yes	scrubbed	5	shift	2	A	1.164
10	before	10	Et.pet	yes	30	yes	untreated	12	regular	2	B	1.172
11	after	8	Et.pet	yes	50	yes	untreated	5	shift	1	B	1.184
12	before	10	Ac.Et	no	30	yes	untreated	5	regular	1	A	1.152

Estimation of the "weight" b_i



Factor	Coefficient
Cst =	1.1629
b_1 =	0.0049
b_2 =	-0.0013
b_3 =	0.0007
b_4 =	0.0101
b_5 =	-0.0002
b_6 =	-0.0029
b_7 =	-0.0021
b_8 =	-0.0013
b_9 =	0.0012
b_{10} =	0.0024
b_{11} =	-0.0008



The quality of the information is better !!!

"1 factor at a time"

X_1 : Threading

before → y_1 → $Var(y_1) = \sigma^2$

after → y_2 → $Var(y_2) = \sigma^2$

22 experiments

$$b_{(threading)} = (y_2 - y_1)/2$$

$$Var[b_{(threading)}] = (\sigma^2 + \sigma^2)/4 = \sigma^2/2$$

Design of experiments $2^{11}//12$

"all the factors together"

b_j

12 experiments

$Var[b_j] = \sigma^2 / 12$

12 experiments

The quality of the information is better !!!

Screening of factors

Example: numerical experiments

Calculation code of Infrared signature *

Objective :

Ranking the factors according to their influence on the dispersion of the signature, considering different scenarios.

The aim of this step is the selection of the most important factors for a later step.

To know the "weight" of each factor

**results from ONERA*

FACTORS :

- Flight altitude
  2 levels
- Flight path
  2 levels
- Atmosphere model
  2 levels
- Day time
  2 levels
- Flight attitude
  3 levels
- Paint emissivity
  3 levels
- Meteo visibility
  3 levels

RESPONSES :

- spectral band II
- spectral band III

To know the "weight" of each factor, for these 2 or 3 levels

Example

Optimal experimental strategy

Experimental design to "weigh"
4 factors with 2 levels and 3 factors with 3 levels,
with an optimal quality and a minimum number of experiments

Optimal asymetrical screening design of experiments

➡ **16 experiments**

Design $2^4 3^3 // 16$

N°	X_1	X_2	X_3	X_4	X_5	X_6	X_7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	1	1	1	3	3	3
4	1	1	1	2	1	1	1
5	2	1	2	1	2	3	1
6	2	1	2	2	1	1	3
7	2	1	2	1	1	1	2
8	2	1	2	2	3	2	1
9	1	2	2	1	3	1	2
10	1	2	2	2	1	3	1
11	1	2	2	1	1	2	1
12	1	2	2	2	2	1	3
13	2	2	1	1	1	2	3
14	2	2	1	2	3	1	1
15	2	2	1	1	2	1	1
16	2	2	1	2	1	3	2

Numerical Experimentation

N°	Flight Altitude	Flight path	Atmosphere model	Time	Attitude	Emissiv. peintures	Meteo. Visibility	Log (Bande II)
1	Alt 1	Path 1	Model 1	Time 1	Att. 1	Emissiv. 1	Meteo 1	-8.260
2	Alt 1	Path 1	Model 1	Time 2	Att. 2	Emissiv. 2	Meteo 2	-8.780
3	Alt 1	Path 1	Model 1	Time 1	Att. 3	Emissiv. 3	Meteo 3	-8.600
4	Alt 1	Path 1	Model 1	Time 2	Att. 1	Emissiv. 1	Meteo 1	-8.460
5	Alt 2	Path 1	Model 2	Time 1	Att. 2	Emissiv. 3	Meteo 1	-8.720
6	Alt 2	Path 1	Model 2	Time 2	Att. 1	Emissiv. 1	Meteo 3	-7.200
7	Alt 2	Path 1	Model 2	Time 1	Att. 1	Emissiv. 1	Meteo 2	-7.760
8	Alt 2	Path 1	Model 2	Time 2	Att. 3	Emissiv. 2	Meteo 1	-8.760
9	Alt 1	Path 2	Model 2	Time 1	Att. 3	Emissiv. 1	Meteo 2	-9.288
10	Alt 1	Path 2	Model 2	Time 2	Att. 1	Emissiv. 3	Meteo 1	-8.740
11	Alt 1	Path 2	Model 2	Time 1	Att. 1	Emissiv. 2	Meteo 1	-8.500
12	Alt 1	Path 2	Model 2	Time 2	Att. 2	Emissiv. 1	Meteo 3	-8.600
13	Alt 2	Path 2	Model 1	Time 1	Att. 1	Emissiv. 2	Meteo 3	-8.300
14	Alt 2	Path 2	Model 1	Time 2	Att. 3	Emissiv. 1	Meteo 1	-10.150
15	Alt 2	Path 2	Model 1	Time 1	Att. 2	Emissiv. 1	Meteo 1	-9.960
16	Alt 2	Path 2	Model 1	Time 2	Att. 1	Emissiv. 3	Meteo 2	-9.140

Interpretation

The flight path is influential

The 2 models are not equivalent

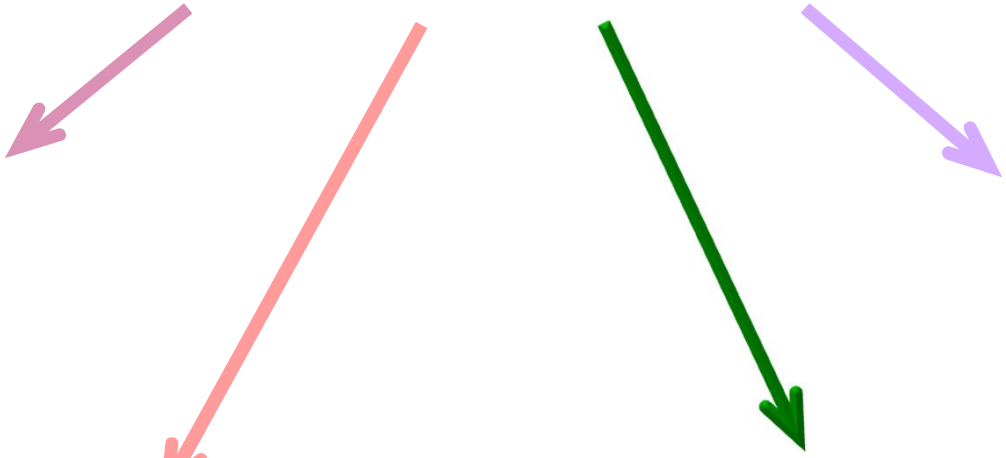
The flight attitude 1 behaves differently from the 2 others

The 3 emissivities are equivalent

To elaborate the experimental strategy

to choose an appropriate design of experiments in accordance with

THE OBJECTIVES



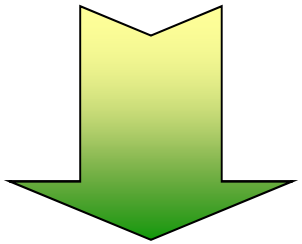
Quantitative study of factors

Possible interactions between the factors

Quantitative study of factors

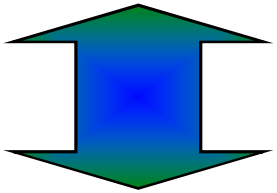
We take into account the possibility of

interaction effects between the factors



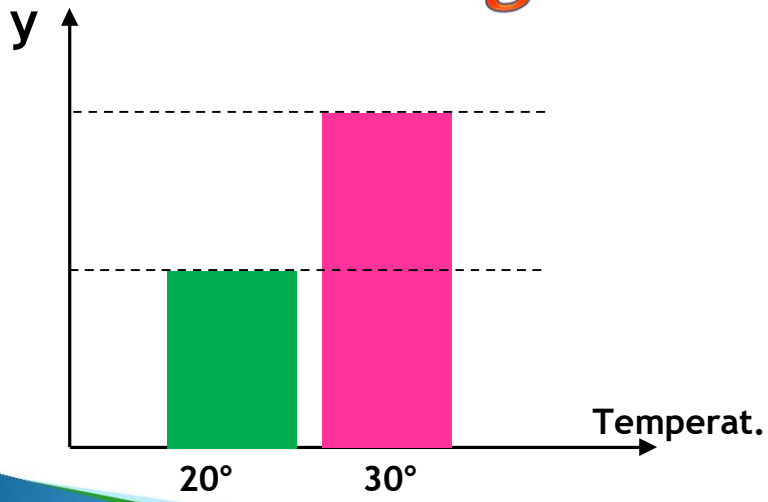
The effect of a factor can be different according to the value of another factor

The effects of the factors X_i et X_j are independant if the effect of the factor X_i doesn't depend on the value of the factor X_j

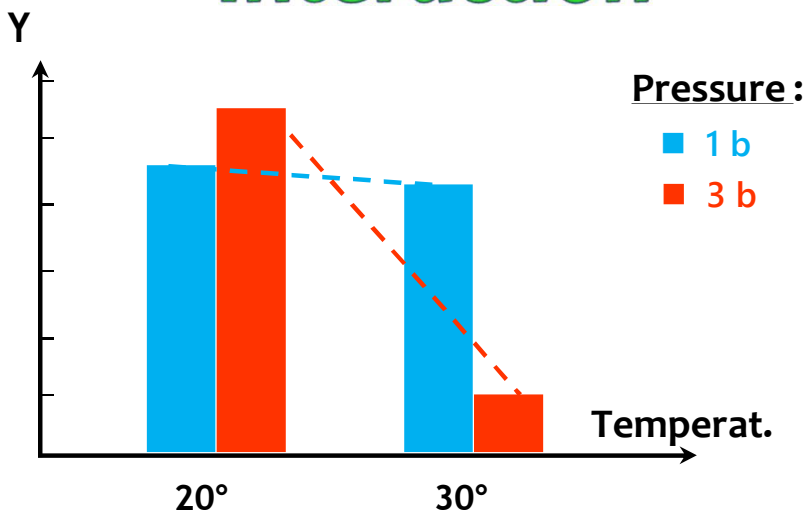


The effects of the factors X_i et X_j are dependant if the effect of the factor X_i depends on the value of the factor X_j

Screening



Interaction



Quantitative study of factors

Sensitivity analysis of interference optical filters*:

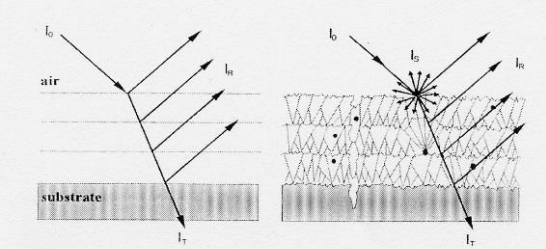
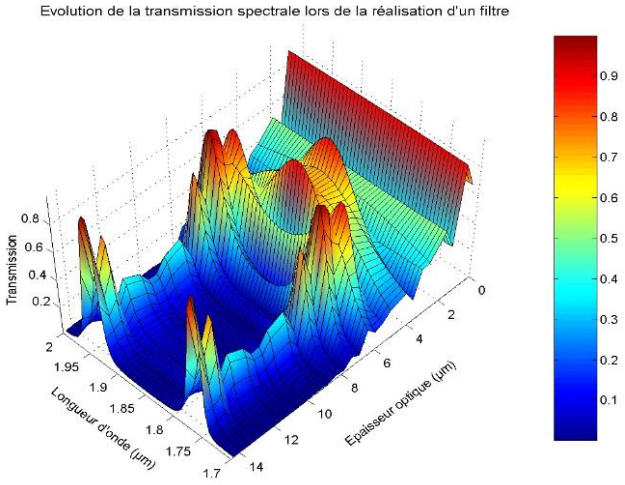


Fig. 1. Left, ideal single-crystalline substrate coated with ideal single-crystalline films. Incoming light with intensity I_0 is split into reflected and transmitted parts I_R and I_T, respectively. Conservation of energy is given by $I_0 = I_R + I_T$. Right, real substrate with real coatings. Part of incoming intensity I_0 is absorbed (I_A) or scattered (I_S). Conservation of energy is given by $I_0 = I_R + I_T + I_A + I_S$.



29-layers optical filter:

Substrate/HLHL4HLHLH L HLHL4HLHLH L HLHL4HLHLH/air

2 materials → 2 refractive index values

H : high refractive index value n_H, L : low refractive index n_L

Desired optical properties → Perfect filter

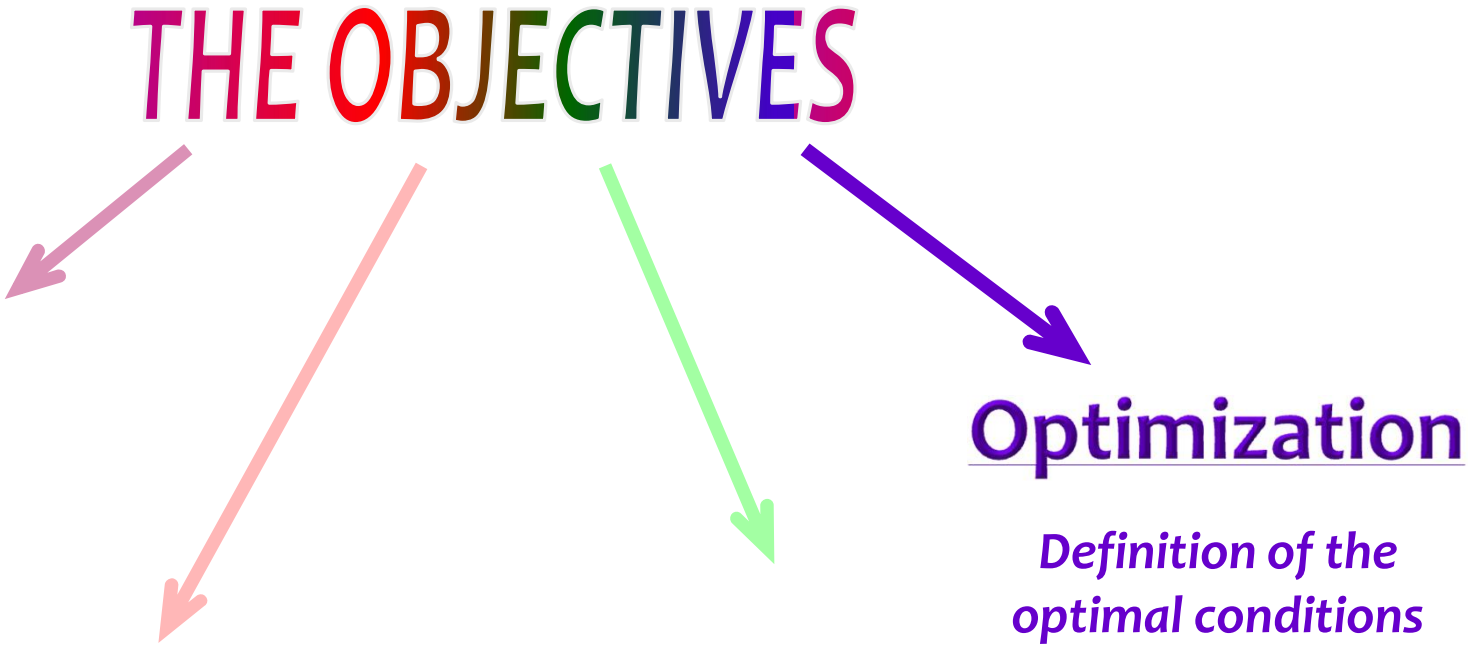
29 parameters:

Variation of refractive index values: +/- x%

*Study realized by O. VASSEUR

To elaborate the experimental strategy

to choose an appropriate design of experiments in accordance with



Quantitative study of the responses :

Optimization

To search for the optimum of one or several responses in the domain of interest

To know, in the whole experimental domain, the value of the experimental response(s)

What is our purpose ?

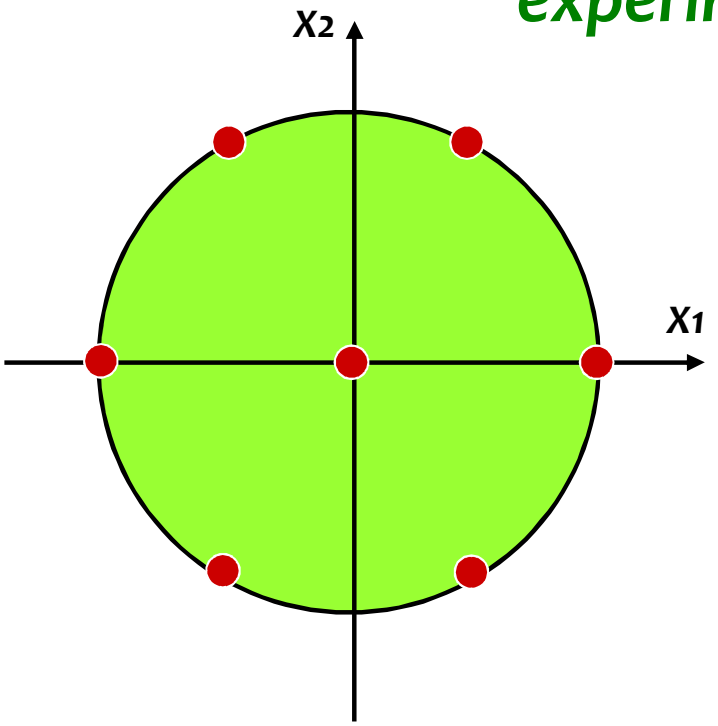
To be able to estimate the value of experimental response in any point within the experimental domain of interest

To find, if it exists, the domain where all the experimental responses respect constraints imposed by specifications

Region of acceptable compromise

We must do experiments

To predict the response value in any point within the experimental domain of interest



Mathematical model

$$Y = f(X_i)$$

Experimental design

If the phenomenon is well represented by the model, *it will be possible to accuratly predict, in any point of the experimental domain, the value of the studied response.*

1) • Postulate an empirical model

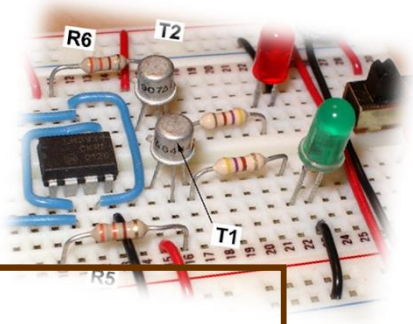
2) • Choose an appropriate experimental design

3) • Run the experiments

4) • Calculate the model

5) • Validation of the model, interpretation

Example : optimization of a process LPVD



FACTORS :

-Temperature TEOS:

62° → 78°C

- Flow TEOS:

55 → 105 ml

- Pressure:

180 → 320 mT

RESPONSES :

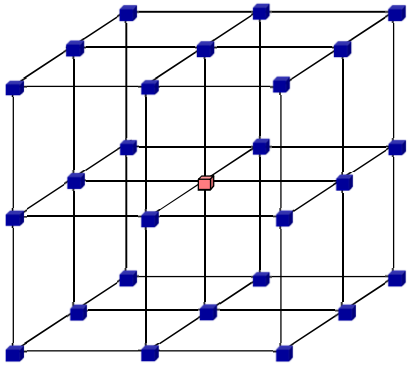
- Run uniformity
- Speed
- Thickness
- Wafer uniformity

To determine the optimal conditions to have the best compromise between the different responses

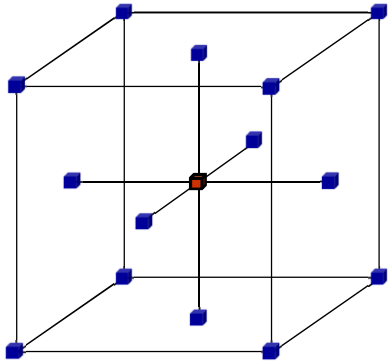
The appropriate experimental design depends on the postulated model

for example,

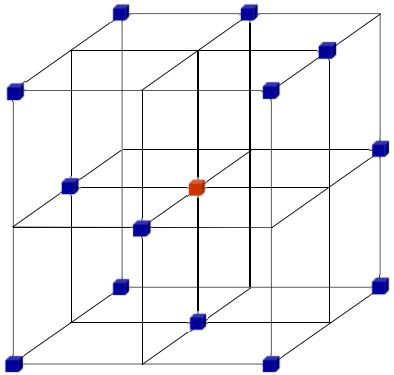
Optimal designs of experiments for a 2nd order polynomial model (3 factors)



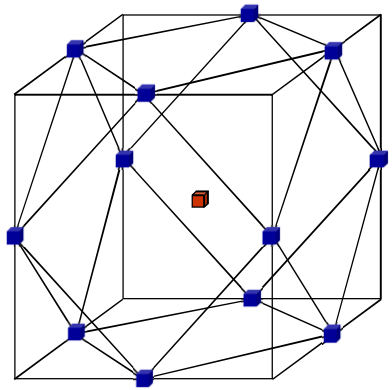
FACTORIAL 3^K



FFC



HOKE D6



BOX-BEHNKEN

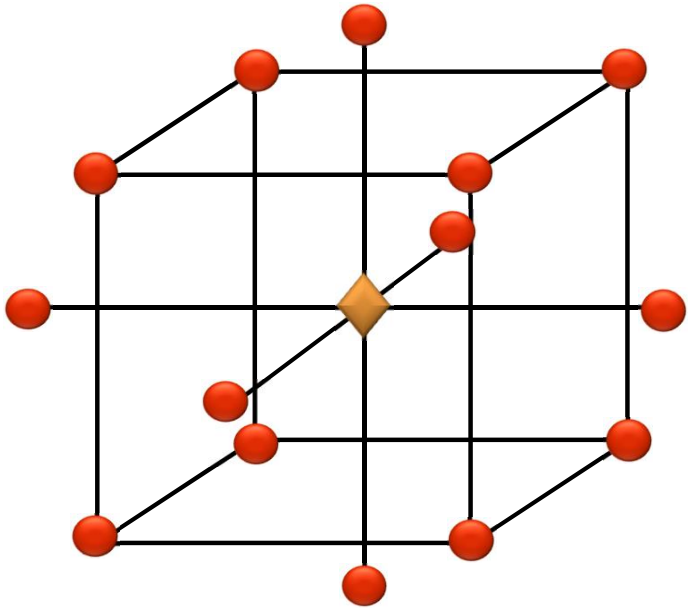
1) • Postulate a model

2nd order polynomial model :

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{11}X_1^2 + b_{22}X_2^2 + b_{33}X_3^2 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3$$

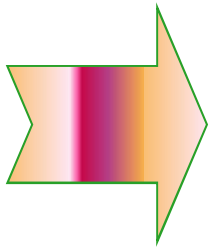
2) • an appropriate experimental design

15 experiments



Experimental design

N°	X1	X2	X3
1	-1,00	-1,00	-1,00
2	1,00	-1,00	-1,00
3	-1,00	1,00	-1,00
4	1,00	1,00	-1,00
5	-1,00	-1,00	1,00
6	1,00	-1,00	1,00
7	-1,00	1,00	1,00
8	1,00	1,00	1,00
9	-1,41	0,00	0,00
10	1,41	0,00	0,00
11	0,00	-1,41	0,00
12	0,00	1,41	0,00
13	0,00	0,00	-1,41
14	0,00	0,00	1,41
15	0,00	0,00	0,00



Experimentation

U1 °C	U2 ml	U3 m T
65	64	200
75	64	200
65	96	200
75	96	200
65	64	300
75	64	300
65	96	300
75	96	300
63	80	250
77	80	250
70	58	250
70	102	250
70	80	179
70	80	320
70	80	250

3)

• Run the experiments

Experimental Responses

N°	Y1	Y2	Y3	Y4
1	3.30	66.90	1120.7	1.64
2	2.89	65.70	1100.9	1.43
3	6.04	59.70	1000.0	2.13
4	6.69	60.23	1008.9	2.23
5	3.64	58.38	977.8	2.29
6	3.65	59.55	997.0	1.55
7	5.36	52.30	876.0	2.52
8	4.80	53.40	894.0	2.09
9	3.93	64.50	1080.0	2.03
10	2.98	54.10	905.0	1.93
11	2.42	63.90	1070.0	1.55
12	6.88	55.10	922.0	2.52
13	3.25	60.58	1015.0	2.10
14	4.72	60.50	1013.0	1.70
15	3.94	60.60	1015.0	1.86

4)

• Estimation of the coefficients

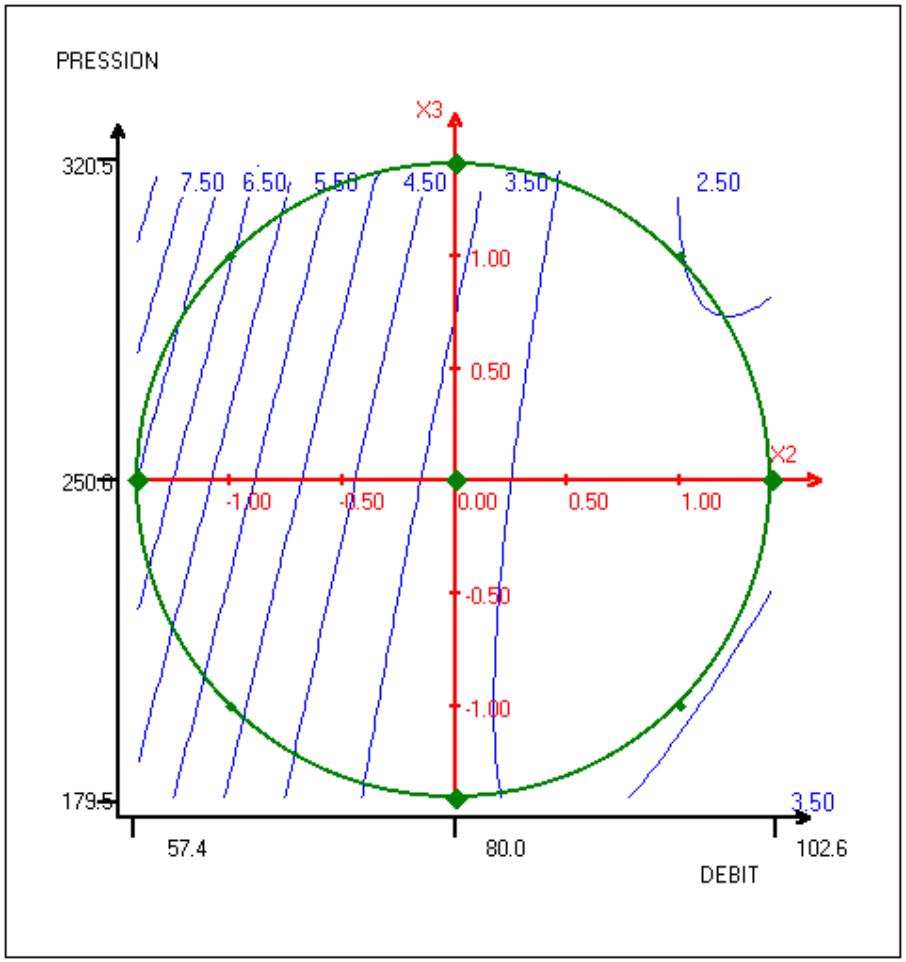
RESPONSE : Run Uniformity (Y_1)

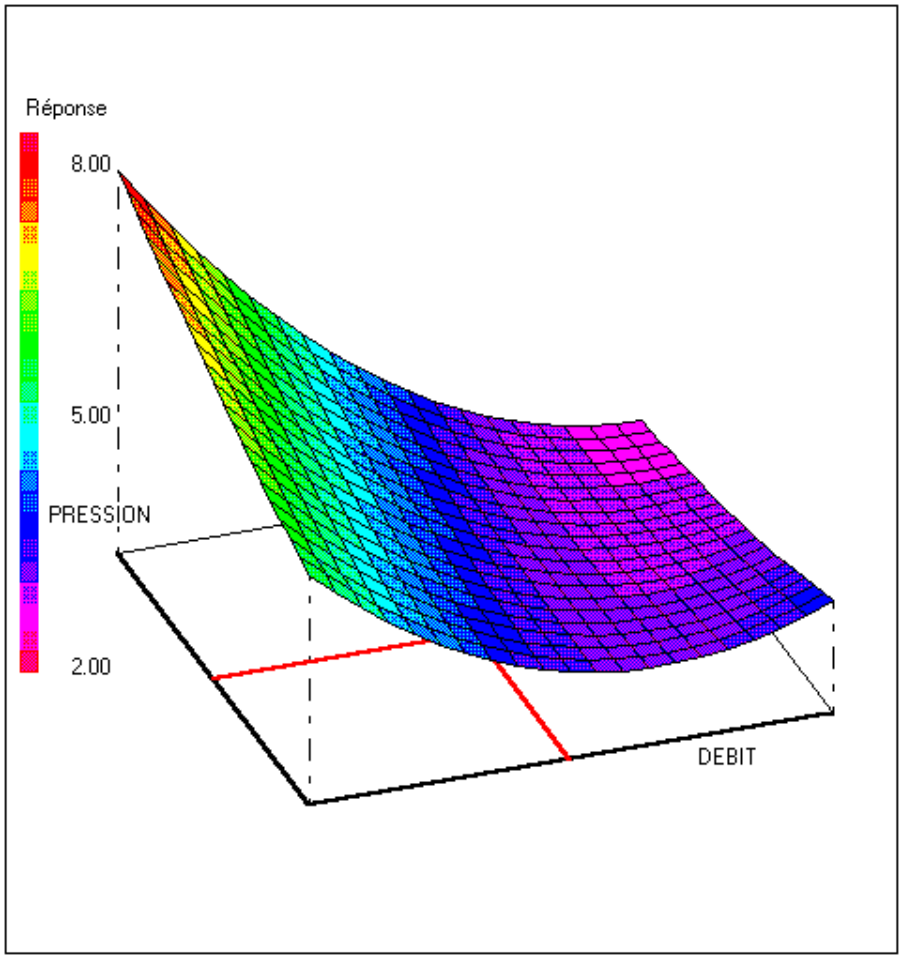
$$Y_1 = 3,28 + 0,12\,X_1 - 1,31\,X_2 + 0,24\,X_3 + 0,51\,X_1^2 + 0,68\,X_2^2 + 0,08\,X_3^2 + 0,06\,X_1 X_2 - 0,10\,X_1 X_3 - 0,46\,X_2 X_3$$

5)

• *Interpretation*

RESPONSE : Run Uniformity (Y_1)

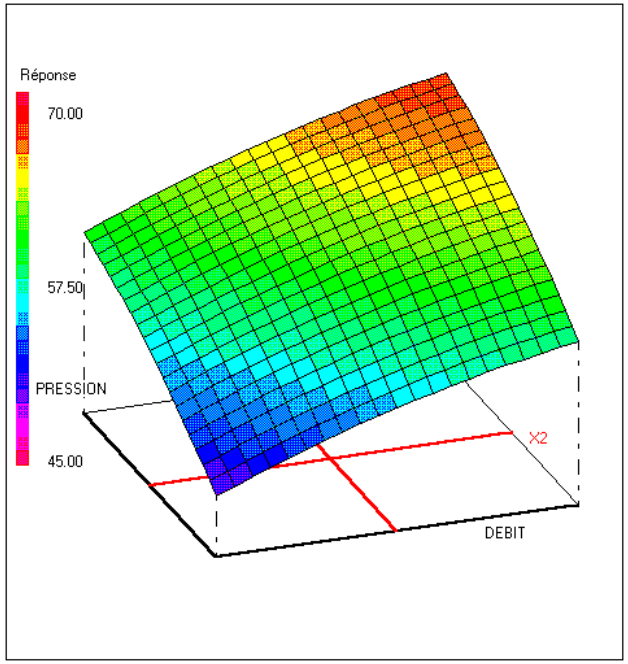




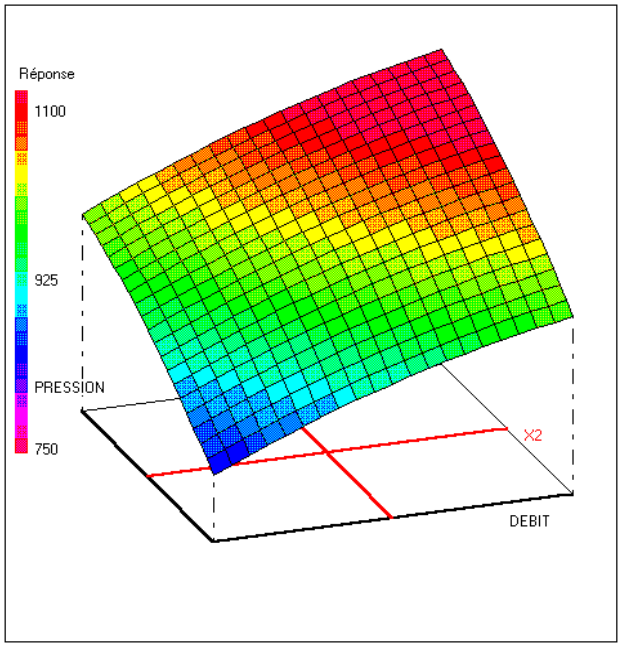
Variation of Uniformity Run in the plane: Flow, Pressure

T°TEOS = 70.0°C

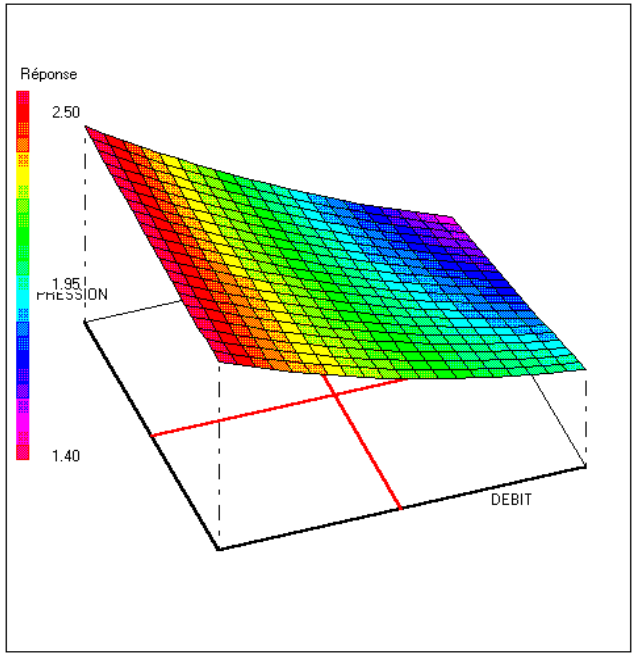
5) • *Interpretation*



Speed (Y_2)



Thickness (Y_3)



Wafer Uniformity (Y_4)

The objectives :

Experimental Responses

η_1 : Run uniformity ≤ 4

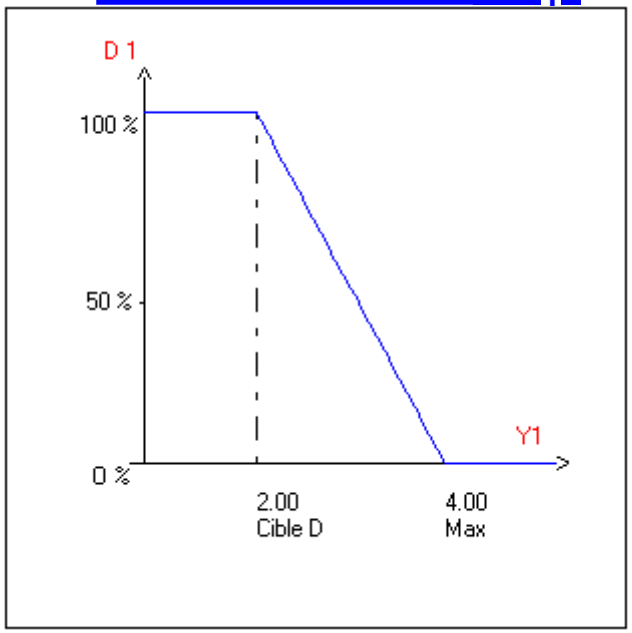
η_2 : $62 \leq$ Speed ≤ 72

η_3 : $950 \leq$ Thickness ≤ 1050

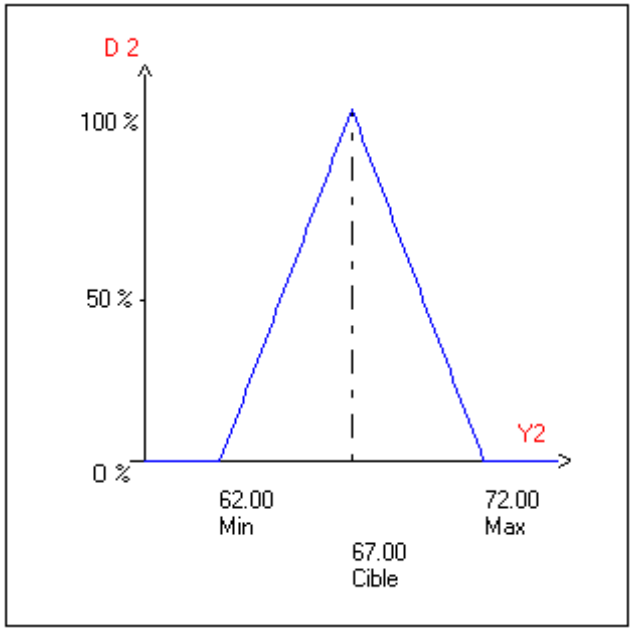
η_4 : Wafer Uniformity ≤ 2

Partial desirability

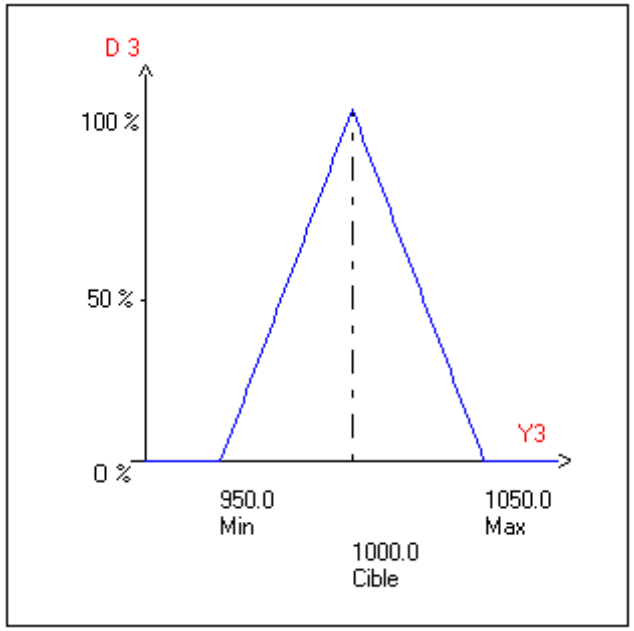
Uniformity Run (Y_1)



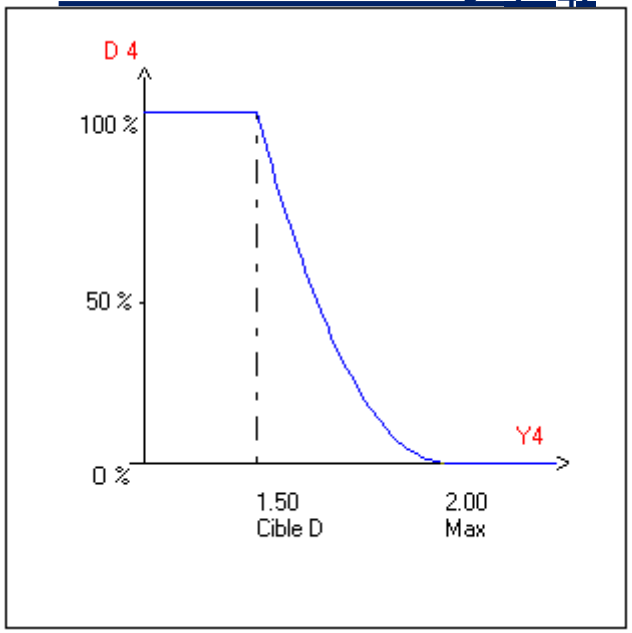
Speed (Y_2)



Thickness(Y_3)



Wafer Uniformity (Y_4)



MULTICRITERIA OPTIMIZATION

Global désirability

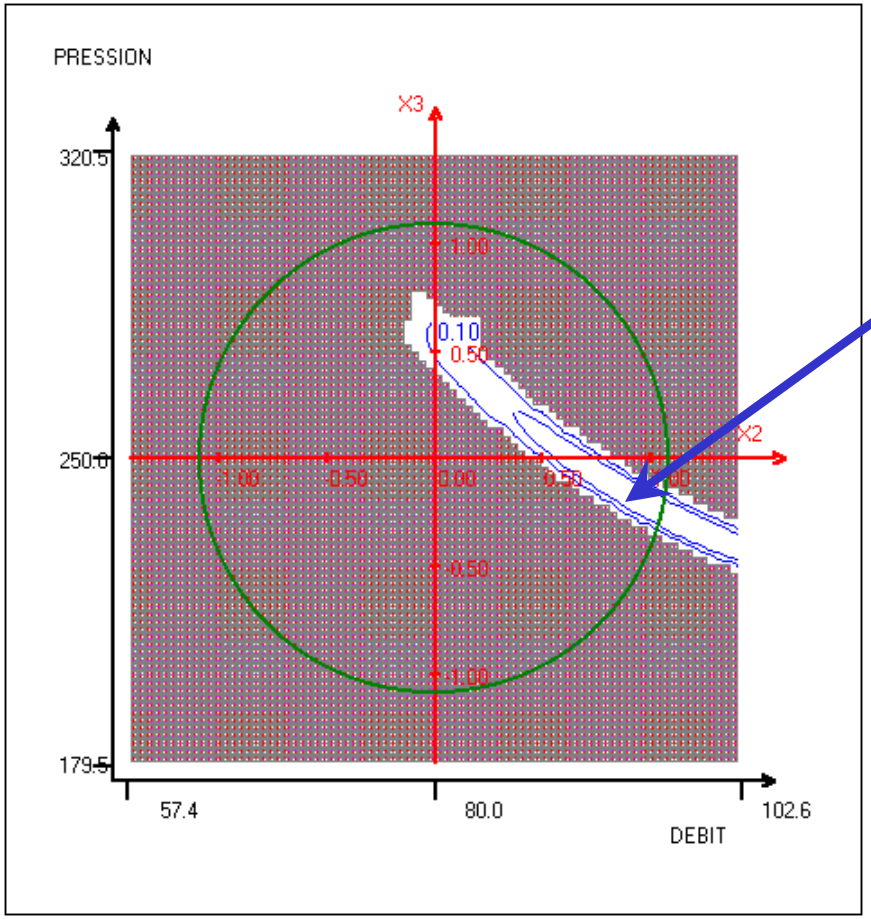
$$D = (d_1^{w1} \times d_2^{w2} \times \ldots\ldots \times d_r^{wr})^{1/W}$$

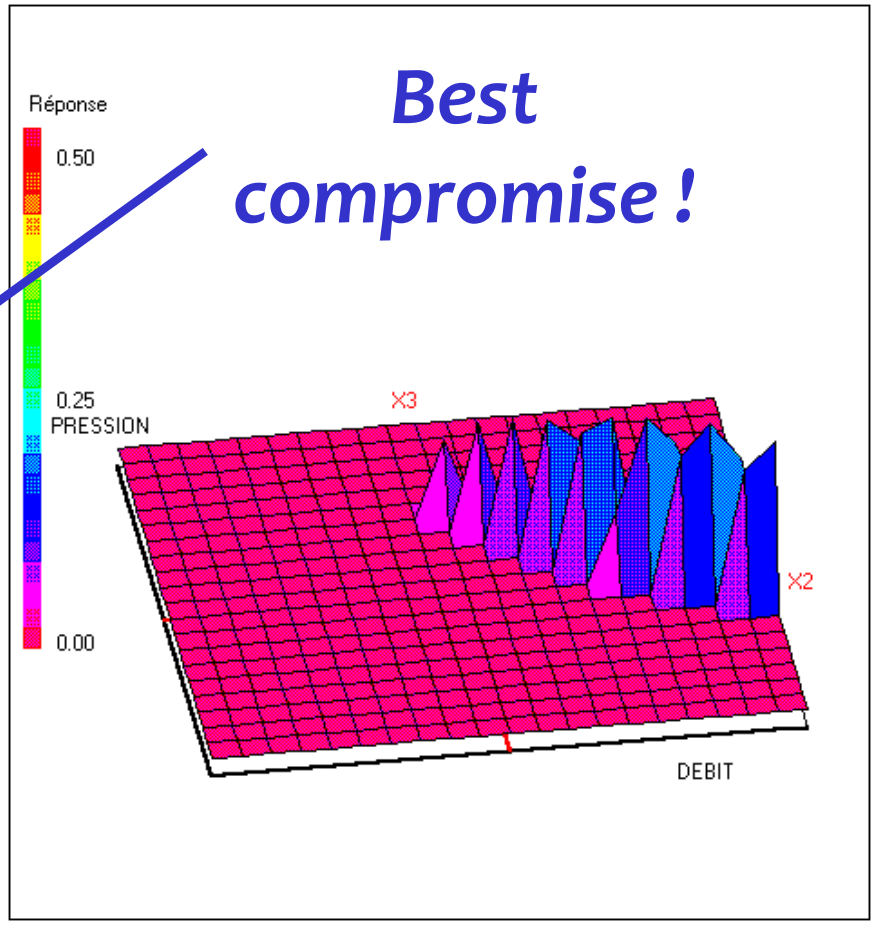
d_j : partial desirability

w_j : reponse ponderation

$$W = \sum w_j$$

Desirability





Best compromise !

Désirability in the plane: Flow, Pressure

T°TEOS = 70.0°C

Designs of experiments

Just tools !!!

Thank you for your attention !

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